On the accuracy and robustness of implicit LES / under-resolved DNS approaches based on spectral element methods

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We present a study on the suitability of under-resolved DNS (uDNS) – also called implicit LES (iLES) – approaches based on spectral element methods (SEM), with emphasis on high-order continuous and discontinuous Galerkin (i.e. CG and DG) schemes. Broadly speaking, these are model-free eddy-resolving approaches to turbulence which solve the governing equations in unfiltered form and rely on numerical stabilization techniques for small-scale regularization. Model problems in 1D, 2D and 3D are used in the assessment of solution quality and numerical stability. A rationale for the excellent potential of these methods for transitional and turbulent flows is offered on the basis of linear dispersion-diffusion analysis.

SEM-based uDNS / iLES approaches have received considerable attention over the last few years (Uranga et al., 2011; Beck et al., 2014; Vermeire et al., 2016; Fernandez et al., 2017). In some cases, these have been shown to outperform traditional LES approaches (even with sophisticated modelling) and provide superior results for the same number of DOFs, see e.g. (Gassner & Beck, 2013; Lombard et al., 2016). However, very few studies have investigated the question of why these methods are able perform so well. At the same time, it is important to explore under which conditions these methods can fail to produce usefully accurate solutions or even “crash” due to under-resolution. As SEM approaches vary considerably depending on discretization variables such as polynomial order, Riemann solver, polynomial dealiasing strategy and alternative stabilization techniques, it is crucial to analyse the effect of these variables on solution quality and numerical stability / robustness.

This study summarizes recent findings concerning why and how to use SEM-based uDNS / iLES and is comprised of three sections. To begin with, the question of DG’s resolution power or eddy-resolving capability is considered. A criterion to estimate DG’s implicit filter width based on linear dispersion-diffusion eigenanalysis (Moura et al., 2015) is discussed and verified in 1D Burgers turbulence simulations. It is shown that higher order DG schemes offer increased resolution power per DOF employed. Secondly, we address the performance of CG with added spectral vanishing viscosity (SVV), see e.g. (Moura et al., 2016), for the simulation of spatially developing vortex-dominated flows in 2D. Linear eigenanalysis is again relied upon for the discussion of the effects of numerical dissipation. An optimization strategy for the SVV parameters is proposed to improve solution quality and robustness. Thirdly, we analyse DG-based 3D computations of the Taylor-Green vortex (TGV) problem at infinite Reynolds numbers (Moura et al., 2017). These results strongly suggest that complete Riemann solvers and moderately high polynomial orders are to be favoured for DG-uDNS / iLES at very high Reynolds. TGV results are also used to verify the resolution power criterion proposed in (Moura et al., 2015), which is confirmed after a simple adaptation (from 1D to 3D settings) is introduced.

REFERENCES