Relative dispersion of particle pairs in turbulent channel flow

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ABSTRACT

The relative dispersion of pairs of Lagrangian particles in a turbulent channel flow at \( Re = 1440 \) is described from direct numerical simulations. Dispersion statistics are conditioned on the initial separation between particles and on their initial wall distance. For all initial configurations, a ballistic growth of the mean square separation is obtained at short times. By tracking particles forwards and backwards in time, the temporal asymmetry of the dispersion process in turbulent channel flows is confirmed. A relative dispersion tensor is introduced and the separation is decomposed in each spatial direction. The tensor analysis gives insight on the preponderant role of the mean shear on the separation of particles, suggesting that the mean shear is responsible for the observed acceleration of the separation rate at intermediate times.

INTRODUCTION

In the Lagrangian framework, a flow is described from the viewpoint of passive tracers moving with the fluid velocity. This approach is particularly well suited for the study of dispersion phenomena such as the transport of pollutants in atmospheric flows. It has been often used to describe turbulent flows in experiments and by direct numerical simulations (Toschi & Bodenschatz, 2009).

The statistical description of the relative dispersion between a pair of passive tracers can be applied to understand the evolution of a particle cloud within a turbulent flow (Salazar & Collins, 2009). The study of particle pair dispersion in statistically isotropic turbulent flows was pioneered by Richardson (1926) and was further developed by Obukhov (1941) and Batchelor (1950). They postulated that the mean square separation of a pair of particles in a turbulent flow follows a succession of diffusive regimes over time, which ultimately depend on the separation distance itself relative to the characteristic scales of turbulent motion. For particle pairs with initial separation \( \mathbf{D}_0 \) is in the inertial range \( (\eta \ll |\mathbf{D}_0| \ll L, \text{where } \eta \text{ and } L \text{ are the Kolmogorov and the integral length scales respectively}) \), the mean squared separation is expected to follow a ballistic regime at short times:

\[
R^2(t) \equiv \langle (\mathbf{D}(t) - \mathbf{D}_0)^2 \rangle = \langle \mathbf{v}_0^2 \rangle t^2 \quad \text{for } t \ll t_0, \tag{1}
\]

where the characteristic time scale \( t_0 \) depends on the initial separation \( \mathbf{D}_0 = |\mathbf{D}_0| \). Here \( \mathbf{D}(t) \) is the instantaneous separation of a particle pair, \( \mathbf{v}_0 \) is their initial relative velocity, and \( \langle \cdot \rangle \) is an ensemble average over a group of particle pairs. The above expression is purely kinematic, and can be readily deduced from the short-time Taylor expansion of \( \mathbf{D}(t) \).

At intermediate times, Richardson (1926) proposed that the relative dispersion process follows a scale-dependent diffusion which leads to

\[
R^2(t) = \varepsilon t^3 \quad \text{for } t_0 \ll t \ll T_L, \tag{2}
\]

where \( T_L \) is the Lagrangian integral time scale, \( \varepsilon \) is the mean turbulent energy dissipation rate and \( g \) is the so-called Richardson constant, supposed universal. This super-diffusive scaling can be derived from Kolmogorov’s (1941) theory using dimensional arguments, assuming particle separations in the inertial range of scales.

Most investigations of relative pair dispersion have been performed in homogeneous isotropic turbulence (Sawford, 2001; Salazar & Collins, 2009). However, realistic flows are generally not isotropic nor homogeneous due to the presence of solid obstacles or boundaries. In this work we aim at describing the dispersion of particle pairs in a turbulent channel flow. This configuration is anisotropic due to the existence of an average shear, and inhomogeneous due to confinement by the walls. Moreover, the fluctuating flow is dominated by the presence of elongated quasi-streamwise vortices near the walls as well as other multi-scale coherent motions...
wall shear stress

(Smits et al., 2011). It is thus of interest to assess the influence of the flow inhomogeneity and anisotropy on the relative dispersion process.

In the following, we present relative dispersion results from direct numerical simulations (DNS) of a turbulent channel flow at a friction Reynolds number $Re = 1440$. The paper is structured as follows. First, the numerical approach is briefly introduced. Then, pair dispersion results are presented and compared to analytical predictions. The influence of the initial pair separation on the dispersion process is then characterised. Finally, a tensorial formulation is introduced and applied to describe the pair separation in different directions.

**NUMERICAL APPROACH**

Direct numerical simulations are performed to study the relative dispersion of fluid particles in a turbulent channel flow between two parallel walls separated by a distance $2h$ as shown in Figure 1. The Reynolds number based on the mean velocity $U_0$ at the channel centre is $Re = U_0 h / \nu = 34,000$, where $\nu$ is the kinematic viscosity. This corresponds to a friction Reynolds number $Re_\tau = u_\tau h / \nu = 1440$, where $u_\tau = \sqrt{\tau_\nu / \rho}$ is the friction velocity associated to the wall shear stress $\tau_\nu$. In the following, the superscript $+$ is used to indicate physical quantities non-dimensionalised by $u_\tau$ and $\nu$.

In the DNS, the Navier-Stokes equations are solved using a pseudo-spectral method (Buffat et al., 2011). The solver is coupled with Lagrangian tracking of tracer particles. The numerical domain is periodic in the streamwise ($x$) and spanwise ($z$) directions, where the solution is decomposed into Fourier modes. In the wall-normal ($y$) direction, a Chebyshev expansion is applied and no-slip boundary conditions are enforced at the channel walls. The domain size is $L_x \times L_y \times L_z = 4\pi h \times 2h \times \pi h$, and the velocity field is decomposed into $2048 \times 433 \times 1024$ spectral modes. The acceleration field is obtained in the Eulerian frame from the resolved velocity $u$ according to $a = \frac{\partial}{\partial t} u + \nabla (u^2 / 2) + (\nabla \times u) \times u$. The velocity and acceleration fields are then interpolated at each particle position using third-order Hermite polynomials.

Dispersion statistics are obtained from two different sets of fluid particles, labeled DS1 and DS2. The dataset DS1 consists of $2 \times 10^6$ particles initially at random positions in the domain. Particle pairs separated by $|D_0| < D_{0\text{max}}$ are identified at wall distances $y^+ = 20, 60, 200, 600$ and 1000 (the channel centre is at $y^+ = 1440$). The maximum separation is taken as $D_{0\text{max}} = 16\eta$, where the Kolmogorov length scale $\eta$ varies with wall distance. Relative separation statistics are obtained by tracking the same particle pairs forwards and backwards in time. This dataset has already been used to study the acceleration of Lagrangian tracers in turbulent channel flow (Stelzennmuller et al., 2017).

The dataset DS2 is initialised at chosen locations in order to characterise the influence on the dispersion process of both the initial position of each pair in the channel, and of their relative initial separation $D_0$. A total of 10 initial wall distances $y^+_0$ ranging from 3 to 1440 are chosen. At each wall distance, particle pairs are given initial separations $D_0 / \eta = 1, 4, 16$ and 64. Moreover, for each initial separation, particle pairs are oriented in each of the 3 Cartesian directions $x, y$ and $z$. This results in 120 different parameter combinations. For each combination, the size of the statistical sample (i.e. the number of particle pairs) is about 20000. Only forward dispersion statistics are obtained from this dataset.

**RELATIVE PAIR DISPERSION**

At short times, the time evolution of the separation between two particles can be described by the Taylor expansion $D(t) = D_0 + \delta v_0 t + \frac{1}{2} \delta a_0 t^2 + \mathcal{O}(t^3)$, where $D_0$ and $\delta a_0$ are the initial relative velocity and acceleration of the pair. This leads to

$$R^2(t) = \langle \delta v_0 \rangle t^2 + \langle \delta v_0 \cdot \delta a_0 \rangle t^3 + \mathcal{O}(t^4) \quad \text{for } t \ll t_0, \tag{3}$$

with $R^2(t) = \langle (D(t) - D_0)^2 \rangle$. The Lagrangian mean $\langle \cdot \rangle$ is defined here as a time-dependent ensemble average over a group of particle pairs that is tracked over time from a given initial condition. Note that Eq. (3) represents a purely kinematic relation and does not take into account the effects of turbulence on the dispersion process. The characteristic time $t_0$ can be interpreted as the memory of the initial condition before the influence of turbulent dispersion.

At the leading order, $R^2$ follows a ballistic regime which depends on the mean square initial relative velocity $\langle \delta v_0^2 \rangle$. If the ensemble average is taken over different occurrences of particle pairs initially located at $(r_0, r_0 + D_0)$, then $\langle \delta v_0^2 \rangle$ is equal to the second-order Eulerian velocity structure function $S_2(r_0, D_0) = \langle u(r_0 + D_0) - u(r_0) \rangle^2$.

At the next order, the $t^3$ term is governed by the crossed velocity-acceleration structure function $\langle \delta v_0 \cdot \delta a_0 \rangle$. Under local homogeneity and stationarity conditions, for spatial increments $D_0$ within the inertial scales, it has been shown that $\langle \delta v_0 \cdot \delta a_0 \rangle = -(\epsilon + \epsilon') = -2\tilde{\epsilon}$, where $\epsilon$ and $\epsilon'$ are the mean turbulent energy dissipation rates at the two probed locations, and $\tilde{\epsilon}$ is the average value between the two (Mann et al., 1999; Hill, 2006). The negative sign indicates that the initial ballistic regime is followed by a deceleration of the dispersion process when going forwards in time. As demonstrated by Jucha et al. (2014), it also implies a temporal asymmetry of the relative dispersion process. From Eq. (3), the difference between forward and backward mean square separation at short times is given by

$$R^2(t) - R^2(-t) = 2\langle \delta v_0 \cdot \delta a_0 \rangle t^3 + \mathcal{O}(t^4). \tag{4}$$
As discussed above, the structure function \( \langle (\delta v_0 \cdot \delta a_0) \rangle \) (in which \( (\delta v_0 \cdot \delta a_0) \) is taken as the time when the second and the third-order terms are important) has been proposed. This characteristic time scale for the short-time dispersion is intrinsic to the short-time dispersion process, since it is negative and independent of the domain. Moreover, its value is close to \(-2\epsilon\) in most of the channel for separations within the inertial range, \( D_0/\eta \approx 16 \) and \( 64 \) in the figure. This observation is confirmed for all the other initial orientations studied here.

In Figure 3, the evolution of the mean square separation is shown for particle pairs of dataset DS1. Forwards and backwards dispersion statistics are represented. A clear initial ballistic regime (in which \( R^2 \) grows as \( t^2 \)) is observed for all wall distances. Following this regime, a growing gap appears between the backwards and forwards dispersion processes, with the former being faster than the latter consistently with the above cited predictions.

In Figure 3, the time evolution of the mean square separation is normalised using wall units. Classically, this evolution can be normalised by a characteristic time scale intrinsic to the short-time dispersion, as in homogeneous isotropic turbulence. In the following, this characteristic time scale for the short-time dispersion is taken as the time when the second and the third-order terms of Eq. (3) are equal in magnitude, i.e. \( t_0 = (\delta v_0^3)/((\delta v_0 \cdot \delta a_0)) \).

As discussed above, the structure function \( (\delta v_0 \cdot \delta a_0) \) is always negative, thus \( t_0 = -\langle (\delta v_0^3) / ((\delta v_0 \cdot \delta a_0)) \rangle \). This characteristic time scale is intrinsic to the short-time dispersion process, since it is derived directly from the analytical expression for the short-time regime. In homogeneous isotropic turbulence, other characteristic time scales for the short-time dispersion have been proposed. Batchelor (1950) postulated that the transition to turbulent dispersion occurs for times larger than the eddy-turnover time at scale \( D_0 = |D_0| \), i.e. \( t_E = D_0^{2/3} \epsilon^{-1/3} \) (Frisch, 1995). In isotropic turbulence, if \( D_0 \) lies within the inertial range, \( t_E \) and \( t_0 \) are equal up to a multiplicative constant (Bourgoin, 2015). This is derived from the \( (\delta v_0 \cdot \delta a_0) = -2\epsilon \) relation along with the inertial range structure function in isotropic turbulence obtained from Kolmogorov’s similarity hypothesis, \( S_2(\delta v_0, D_0) = S_2(D_0) = C (\epsilon D_0)^{5/3} \), where \( C \) is a universal constant.

A comparison between \( t_0 \) and \( t_E \) for different initial separations and initial wall distances is presented in Figure 4. The initial orientation of the separation is along \( z \). Also shown is the time scale \( t_0^* = (\delta v_0^3) / (2\epsilon) \) which is obtained by assuming that \( (\delta v_0 \cdot \delta a_0) \) is roughly \(-2\epsilon\) in the definition of \( t_0 \). The dataset DS2 is used here. In Figure 4, the time scales \( t_0 \) and \( t_0^* \) match for initial separations \( D_0/\eta \approx 16 \) and \( 64 \) in most of the channel (\( y^+ > 20 \)), confirming that under those conditions the identity \( (\delta v_0 \cdot \delta a_0) = -2\epsilon \) holds, as shown in Figure 2. For smaller \( y^+ \), the differences may be due to the increasing energy dissipation rate, which peaks at \( y^+ = 0 \) (Pope, 2000). On the other hand, the identity is never recovered for small initial separations, \( D_0/\eta \approx 1 \) and \( 4 \), since these two initial separations are not within the inertial scales. The chosen time scale \( t_0 \) behaves similarly to the eddy-turnover time \( t_E \). For the smallest separations \( D_0/\eta \approx 1 \) and \( 4 \), the two characteristic times evolve as two parallel curves for all wall distances. For the largest separations, their slopes noticeably differ in the near-wall region up to \( y^+ \approx 30 \).

In Figure 5, the forwards dispersion curves of Figure 3 are shown scaled by the characteristic time \( t_0 \) and by the Eulerian velocity structure function \( (\delta v_0^3) \). Under this scaling, curves associated to different wall distances collapse for times up to \( t \approx 2t_0 \), emphasising the relevance of the proposed scaling. A remarkable \( t^3 \) ballistic regime is also observed for all wall distances. This suggests that for short-time pair dispersion stochastic models, purely kinematic considerations would be appropriate, even in inhomogeneous and anisotropic turbulent conditions.

Figure 6 plots the local scaling exponent of the pair separation process obtained from DS1, i.e., the local slopes of the curves shown in Fig. 3. The time is normalised by \( t_0 \). In both forwards and backwards dispersion, a slight departure from the pure ballistic regime is found as early as \( |r|/t_0 \approx 0.01 \). Consistently with Fig. 3 and with the analytical predictions, this departure is given by an increased separation rate in the backwards dispersion process, and a
When the separation distance is larger than the integral length scale, the mean separation rate that ends with a peak. Excepting the smallest wall crossing, the scaling exponent at its peak increases with wall distance, and it even of an exact phase corresponds to Richardson’s normalised by the mean square separation.

\[ \langle (D(t) - D_0)^2 \rangle / \langle (\delta v_0 \cdot \delta a_0) \rangle^2 \]

Figure 5. Forward mean square separation, normalised by the structure function \( \langle \delta v_0^2 \rangle \) and the characteristic time \( t_0 = -\langle \delta v_0^2 \rangle / \langle \delta v_0 \cdot \delta a_0 \rangle \). Colours represent the initial wall distance of each pair. Results are obtained from dataset DS1.

At long times, the mean separation rate decreases monotonically. At intermediate times, all cases present a phase of increasing threshold in some cases. Moreover, in this time interval, the existence of an exact \( t^3 \) regime is recovered for all wall distances including the near-wall region \( (y^+ = 20) \). The results are consistent with the experimental measurements and the DNS of Jucha et al. (2014) in isotropic turbulence.

At intermediate times, all cases present a phase of increasing separation rate that ends with a peak. Excepting the smallest wall distance \( y_0^+ = 20 \), the peak is located at \( 2 < |r| / t_0 < 5 \). Assuming that this phase corresponds to Richardson’s \( t^3 \) regime, the existence of an exact \( t^3 \) scaling might be questioned here. The value of the scaling exponent at its peak increases with wall distance, and it even crosses the \( t^3 \) threshold in some cases. Moreover, in this time interval, forward dispersion evolves faster than backward dispersion.

At long times, the mean separation rate decreases monotonically. When the separation distance is larger than the integral length scale \( L \) associated to the largest turbulent structures, the motion of particles in a pair can be expected to become completely decorrelated. In this case, their separation rate would correspond to a normally diffusive process represented by a linear evolution of \( R^2 \).

The difference between forward and backward dispersion is shown in Figure 7, compensated by \( \langle \delta v_0 \cdot \delta a_0 \rangle^2 \). At short times, the prediction of Eq. (4) is recovered for all wall distances including the near-wall region \( (y^+ = 20) \). The results are consistent with the experimental measurements and the DNS of Jucha et al. (2014) in isotropic turbulence.

**INFLUENCE OF THE INITIAL SEPARATION**

As described in the previous section, the short-time dispersion of a pair of particles is governed in average by the second-order Eulerian velocity structure function \( S_2 \) evaluated at the separation \( D_0 \). In isotropic turbulence, \( S_2 \) increases with the separation (as \( D_0^{2/3} \)) when \( D_0 = |D_0| \) is within the inertial range. In wall-bounded turbulence, \( S_2 \) not only depends on the separation magnitude \( D_0 \), but also on the orientation of \( D_0 \) and on the probed wall distance \( y_0^+ \).

In particular, the structure function is largely anisotropic near the walls due to the strong mean shear and the influence of coherent structures on the fluctuating velocity field. This section briefly presents relative dispersion results for particles initially located very near the wall.

In Figure 8, the mean square separation is shown for particle pairs initially located at \( y_0^+ = 8 \). Results correspond to dataset DS2. Two initial separations are compared, \( D_0 / \eta = 1 \) and 64, along with the three Cartesian directions as initial orientations. The short-time ballistic behaviour is clearly observed for all three initial orientations and both initial separations. The rate of evolution of the ballistic regime, determined by the velocity structure function \( S_2 \), differs for each orientation and each initial separation. In general, the rate of evolution is weaker for the smallest separations (\( D_0 / \eta = 1 \)), since the velocity field does not vary significantly in space at separation scales similar to that of the smallest turbulent eddies, and thus the relative velocity \( \delta v_0 \) is weaker for those separations.

As expected, the initial orientation of the pairs plays an important role very close to the wall. This is associated with a strongly anisotropic velocity structure function in that region. Near the wall, the velocity field is most correlated in the streamwise direction due to the preferential orientation of coherent structures in that direction, e.g. quasi-streamwise vortices, sweeps and ejections (Robinson, 1991; Panton, 2001). For a given initial separation, a higher correlation coefficient is linked to a weaker structure function \( S_2 \) and both initial separations. The rate of evolution of the ballistic behaviour is clearly observed for all three initial orientations and both initial separations. The rate of evolution of the ballistic regime, determined by the velocity structure function \( S_2 \), differs for each orientation and each initial separation. In general, the rate of evolution is weaker for the smallest separations (\( D_0 / \eta = 1 \)), since the velocity field does not vary significantly in space at separation scales similar to that of the smallest turbulent eddies, and thus the relative velocity \( \delta v_0 \) is weaker for those separations.

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Therefore, the velocity structure function is expected to be the weakest for streamwise separations. This is confirmed by the curves in Figure 8, showing a separation rate that is weaker in the streamwise direction at short times. The opposite occurs in the wall-normal direction. The mean shear has a predominant contribution in $S_z$ for wall-normal separations. Consequently, the rate of separation in the ballistic regime is the highest when particles are oriented in $y$.

As can be expected, at long times the mean separation no longer depends on the initial configuration of the pairs. An intermediate time range connecting the short-time ballistic dispersion, which is strongly dependent on the initial separation, with the long-time dispersion regime, independent of the initial pair separation, can be observed. The rate of separation in the intermediate regime appears to be determined by the gap between the ballistic and the long-time regimes. Ultimately, since the long-time separation rate is the same for all initial configurations, a slow separation at short times (associated with a weak value of $S_z$) is typically followed by a fast separation at intermediate times.

**RELATIVE DISPERSION TENSOR**

To better understand the effect of inhomogeneity on relative dispersion, it is convenient to analyse the separation of particles independently in each direction. More generally, it is possible to define a relative dispersion tensor $\Delta$ that can also describe the interdependency between separation components:

$$\Delta_{ij}(t) = \langle (D_i(t) - D_0)(D_j(t) - D_0) \rangle,$$

with $(i, j) \in \{x, y, z\}$. Here $D_i$ and $D_0$ are the $i$-th component of the separation vectors $D$ and $D_0$, respectively. The total mean square separation can be recovered as the trace of this tensor, $R^2(t) = \Delta_{ii}(t)$. Note that $\Delta$ is symmetric. Moreover, the non-diagonal components $\Delta_{xz}$ and $\Delta_{yz}$ are zero due to the statistical symmetry $z \leftrightarrow -z$. Thus, the only non-zero non-diagonal component is $\Delta_{xy} = \Delta_{yx}$.

The short-time behaviour derived for $R^2(t)$ in Eq. (3) can be generalised to:

$$\Delta_{ij}(t) = \langle \delta v_{0i} \delta v_{0j} \rangle t^2 + \left( \langle \delta v_{0i} \delta a_{0j} \rangle + \langle \delta v_{0j} \delta a_{0i} \rangle \right) t^3 + O(t^4).$$

Analogously to the short-time evolution of $R^2$, each component $\Delta_{ij}$ independently follows a ballistic regime at short times, with a growth rate given by a component of the second-order Eulerian velocity structure function tensor $S_{ij}(r_0, D_0) = \{u_i(r_0 + D_0) - u_i(r_0)\} \{u_j(r_0 + D_0) - u_j(r_0)\} = \langle \delta v_{0i} \delta v_{0j} \rangle$. The third-order term that follows the ballistic growth is governed by the symmetric part of the crossed velocity-acceleration structure function tensor $\langle \delta v_{0i} \delta a_{0j} \rangle$.

In Figure 9, the evolution of the non-zero terms of the dispersion tensor are shown for particle pairs initially located at $y_0^+ = 67$ obtained from dataset DS2. Pairs are initially separated in the spanwise direction by a distance $D_0 = 16\eta$, which corresponds to $D_0^+ = 37$ at the chosen wall distance. Because of this initial orientation, the mean flow does not change between the initial positions of the two particles in a pair. Therefore, the mean shear is not expected to play an important role in the short-time dispersion process. In Figure 9, the short-term ballistic regime is observed for all the components of the dispersion tensor. At the chosen wall distance and for the initial pair separation presented here, the $S_{xy}$ component of the velocity structure function tensor is higher than for the other components.

At intermediate times, the diagonal components of $\Delta$ present different behaviours. Starting from $t/t_0 \approx 0.6$, $\Delta_{xx}$ has an increasing rate of separation, meaning that particle pairs separate faster in the streamwise direction. This is not the case for the wall-normal and spanwise components, which do not present a visible acceleration of the separation process at intermediate times. On the contrary, the separation rate in those directions slows down relative to the initial ballistic regime. For times larger than $t \approx 2t_0$, the curves for $\Delta_{xx}$ and $R^2$ match, implying that the total mean square separation is determined almost uniquely by the contribution of the streamwise separation (i.e. $R^2 \approx \Delta_{xx} \gg \Delta_{yy}, \Delta_{zz}$). This relative importance of the streamwise component is explained by the effect of the mean shear, which becomes dominant when the wall-normal particle separation is sufficiently large.

In the range $1 < t/t_0 < 4$, the streamwise diffusive regime scales with $r^3$, which is comparable to Richardson’s $r^3$ regime. This is not observed for the mean square separation $R^2$, which presents a slower scaling due to the contribution of the slower-evolving components $\Delta_{yy}$ and $\Delta_{zz}$.

At intermediate and long times, particles separate slower in the
confinement by the walls, which enforce an upper bound for the wall-normal particle separation. This bound is given by $|D_y(t)| < 2h$, where $D_y$ is the wall-normal component of the particle separation and $2h$ is the channel width. In terms of the dispersion tensor, this corresponds to $\Delta_{yy}(t) \leq (2h + |D_{0y}|)^2 \approx (2h)^2$ if $|D_{0y}| \ll h$. The level $\Delta_y = (2h)^2$ is shown in Fig. 9 for reference.

At long times, $\Delta_{xy}$ and $\Delta_{yz}$ follow a linear evolution in time, associated with a normally diffusive regime in which the particle separation is large enough for the respective particle motions to be uncorrelated. The streamwise component (as well as $R^2$) tends to a normally diffusive regime, but much more slowly than the other two diagonal components.

The cross term $\Delta_{xy}$ is negative during the ballistic regime (i.e. the structure function $S_{xy}$ is negative). The departure from the ballistic regime is given by an evolution towards positive values. This can be explained by a positive contribution of the third-order term in Eq. (6). It is worth noting that, although its initial value is negative, $\Delta_{xy}(t)$ is an increasing function at all times. The cross term becomes positive at $t/t_0 \approx 0.5$. At intermediate times, it undergoes a phase of fast growth similar to that of $\Delta_{xz}$. Later, its time evolution decelerates and becomes similar to that of $\Delta_{xy}$ and $\Delta_{xz}$ at long times, with a growing rate that appears to be linear.

The positive sign of $\Delta_{xy}$ at sufficiently long times can be explained as an effect of the mean shear. Let us consider a pair of particles $(A,B)$ separated by $D = D^y - D^z$, initially located in the lower half of the channel ($y_0 < h$). For simplicity, let us suppose that, as in Figure 9, their initial separation vector is normal to the wall-normal direction, so that the mean shear is not important at short times. Without loss of generality, it can be assumed that at some point, particle $B$ is further away from the wall than particle $A$, i.e. $D_y = y^B - y^A > 0$, and this distance becomes large enough for mean shear effects to be important (at small scales, turbulent motion may play a more important role than the average flow in the separation process). As a consequence of the mean shear, the streamwise velocity of particle $B$ is then larger in average than that of particle $A$, leading to a relative streamwise velocity $\delta v_z = \frac{dD_y}{dt} = v^B - v^A > 0$, and consequently to a rapidly increasing streamwise separation $D_z$. The result is a coefficient $D_y D_z$ that is an increasing function of time for as long as $D_z$ remains positive.

**CONCLUDING REMARKS**

The relative dispersion of pairs of fluid particles in a turbulent channel flow is described from direct numerical simulations. The expected short-time behaviour for the mean square particle separation, corresponding at leading order to a ballistic growth in time, is recovered for all the explored initial separations and wall distances. The temporal asymmetry of the short-time dispersion process in channel flow is verified, in agreement with similar results in homogeneous isotropic turbulence (Jucha et al., 2014). By decomposing pair separation into a relative dispersion tensor, it is confirmed that the diffusion is governed by the mean shear in the channel at intermediate and long times. It is suggested by the tensor analysis that the mean shear is responsible for a regime of accelerated streamwise separation in which the mean square separation evolves as $t^3$. It remains to be determined whether a similar behaviour exists for backwards dispersion at intermediate and long times. The results presented here will further be used to test simple kinematic models for particle pair dispersion (Bourgoin, 2015) or for determining parameters in existent two-particle stochastic models (Durbin, 1980; Sawford, 2001; Borgas & Yeung, 2004).

**REFERENCES**


